Pressure Drop Through Tapered Dies

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Synopsis

A simple method is presented for calculating the pressure drop for the flow of power law liquids in dies with a wide slit profile and with vertical and/or lateral tapers, as well as in dies with the shape of a circular truncated right cone. Tapered dies are known to give improved extrudate quality and/or higher output rates without encountering extrudate defects which occur in dies with parallel channels at similar extrusion pressures. A possible ultimate optimization of the extrusion process—as far as die design is concerned—is discussed. It is suggested that this be based upon an extension of the method from dies with a rectilinear convergent taper to dies with a curvilinearly converging channel aspect the generation of which latter is indicated.

INTRODUCTION

When extruding polymer melts, the use of tapered dies with taper angles of less than 10° is found to be beneficial in preventing flow defects at operating pressures which are substantially greater than those which would cause serious defects in dies with parallel-sided channels. Attention to this has been drawn by Cogswell and Lamb[1] who also gave a mathematical treatment of the problem. A similar mathematical treatment has been given by Plajer.[2]

The present work gives a more rigorous treatment which is, at the same time, remarkably simple.

DERIVATION OF THE PRESSURE DROP EQUATIONS

We consider four typical and common examples of linearly convergent dies, namely: (i) vertically tapered wide-slit dies; (ii) laterally tapered wide-slit dies; (iii) wide-slit dies with a vertical *as well as* a lateral taper; and (iv) dies having the shape of a circular truncated right cone.

Pressure Drop Through a Wide-Slit Die of Constant Width w and with a Vertical Taper Angle θ

A diagram of the geometry is given in Figure 1. It is seen that

$$h = H_1 = l \tan \theta.$$

$$\tan \theta = (H_1 - h)/l = -dh/dl$$

$$dh = -dl \tan \theta$$

$$dl = -dh \cot \theta.$$

In wide-slit dies,

$$\dot{\gamma} = \frac{2n+1}{3n}\dot{\gamma}_{\text{app}} = \frac{2n+1}{3n}\cdot\frac{6Q}{wh^2} = \frac{Q}{wh^2}\cdot\frac{4n+2}{n}.$$

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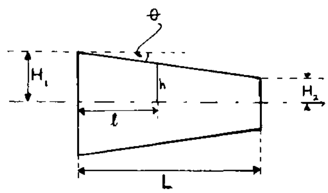

Fig. 1. Median longitudinal section through tapered die channel.

(Note the use of the appropriate Rabinowitsch correction for power law liquids in wide-slit channels.) In flow through a slit die *without* taper,

$$\tau = \frac{\Delta P h}{2L} = \eta \dot{\gamma}^n = \eta \left(\frac{Q}{wh^2} \cdot \frac{4n+2}{n}\right)^n$$

whence

$$\Delta P = \frac{2L\eta}{w^n h^{2n+1}} \left(Q \frac{4n+2}{n}\right)^n \qquad \text{(A)}$$

If the wide-slit die is vertically tapered, we consider the pressure drop dP for an infinitely small length dl in terms of dh and integrate with respect to h between the limits of H_1 and H_2, remembering that $dl = f(dh)$, namely,

$$dl = -dh \cot \theta$$

$$dP = \frac{2dl\eta}{h^{2n+1}} \left(Q \frac{4n+2}{wn}\right)^n$$

$$\Delta P = -2\eta \cot \theta \left(Q \frac{4n+2}{wn}\right)^n \int_{H_1}^{H_2} h^{-(2n+1)}\, dh$$

$$= -2\eta \cot \theta \left(Q \frac{4n+2}{wn}\right)^n \left(-\frac{1}{2n}\right) h^{-2n} \Big|_{H_1}^{H_2}$$

Simplifying and taking the limits, we get

$$P = \frac{\eta \cot \theta}{n} \left(Q \frac{4n+2}{wn}\right)^n H_2^{-2n} \left[1 - \left(\frac{H_1}{H_2}\right)^{-2n}\right] \qquad (1)$$

Pressure Drop Through a Wide-Slit Die with Converging Sides

This is an inverted fishtail die with bilaterally equal sideways taper angle ϕ and with constant height h. The width w reduces from the entrance width w_1 to the exit width w_2.

By trigonometry, as before,

$$\tan \phi = -\frac{dw}{dl}$$

$$dw = -dl \tan \phi$$

$$dl = -dw \cot \phi$$

Starting from eq. (A), we consider the pressure drop dP for an infinitely small length dl in terms of dw and integrate with respect to w between the limits of w_1 and w_2, remembering that $dl = f(dw)$, namely,

$$dl = -dw \cot \phi$$

$$dP = 2\eta h^{-(2n+1)} \left(Q \frac{4n+2}{n}\right)^n dw \cot \phi \, w^{-n}$$

$$P = -2\eta h^{-(2n+1)} \left(Q \frac{4n+2}{n}\right)^n \cot \phi \int_{w_1}^{w_2} w^{-n} \, dw$$

$$= \frac{2\eta}{n-1} h^{-(2n+1)} \left(Q \frac{4n+2}{n}\right)^n \cot \phi \, w^{1-n} \Big|_{w_1}^{w_2}$$

Simplifying and taking the limits,

$$\Delta P = \frac{2\eta}{n-1} h^{-(2n+1)} \left(Q \frac{4n+2}{n}\right)^n \cot \phi \, w_2^{1-n} \left[1 - \left(\frac{w_1}{w_2}\right)^{1-n}\right] \quad (2)$$

Pressure Drop Through an Inverted Fishtail Die with Lateral Taper Angle ϕ and with a Simultaneous Vertical Taper Angle θ

By trigonometry, as before,

$$\tan \phi = -\frac{dw}{dl}$$

$$dw = -dl \tan \phi$$

$$dl = -dw \cot \phi$$

Starting from eq. (A), we consider the pressure drop dP for an infinitely small length dl in terms of dh and dw and integrate with respect to (i) h between the limits of H_1 and H_2, (ii) w between the limits of w_1 and w_2, remembering that dl may be expressed as $f(dh)$ *and* as $f(dw)$, namely,

$$dl = -dh \cot \theta$$

and

$$dl = -dw \cot \phi$$

$$dP = 2\eta h^{-(2n+1)} \left(Q \frac{4n+2}{n}\right)^n (-dh \cot \theta) w^{-n} (-dw \cot \phi)$$

and

$$\Delta P = 2\eta \cot \theta \cot \phi \left(Q \frac{4n+2}{n}\right)^n \int_{H_1}^{H_2} h^{-(2n+1)} \, dh \int_{w_1}^{w_2} w^{-n} \, dw$$

Integration yields

$$P = 2\eta \cot \theta \cot \phi \left(Q \frac{4n+2}{n}\right)^n \left(-\frac{1}{2n}\right) \left(-\frac{1}{n-1}\right) h^{-2n} w^{1-n}.$$

Taking the limits and simplifying, we finally obtain

$$\Delta P = \frac{\eta \cot\theta \cot\phi}{n(n-1)} \left(Q \frac{4n+2}{n}\right)^n \times H_2^{-2n} \left[1 - \left(\frac{H_1}{H_2}\right)^{-2n}\right] w_2^{1-n} \left[1 - \left(\frac{w_1}{w_2}\right)^{1-n}\right] \quad (3)$$

Pressure Drop Through Dies Having the Shape of a Circular Truncated Right Cone

The geometry of this die may be represented, mutatis mutandis, by a diagram similar to the one applicable to flow in a vertically tapered wide-slit die (see above). In the present case, however, h, H_1, and H_2 are replaced by r, R_1, and R_2 respectively, and it is also understood that the taper angle θ is operative around the entire circumference of the die. We can therefore write

$$r = R_1 - l \tan\theta$$

$$\tan\theta = (R_1 - r)/l = -dr/dl$$

$$dr = -dl \tan\theta$$

$$dl = -dr \cot\theta.$$

In circular dies,

$$\dot{\gamma} = \frac{3n+1}{n} \dot{\gamma}_{\text{app}} = \frac{3n+1}{n} \frac{4Q}{\pi R^3}.$$

(*Note:* The inclusion of the Rabinowitsch correction distinguishes this treatment from that given by Cogswell and Lamb.)

In flow through an *untapered* circular channel,

$$\tau = \frac{\Delta P R}{2L} = \eta \dot{\gamma}^n = \eta \left(\frac{4Q}{R^3} \frac{3n+1}{\pi n}\right)^n$$

and

$$\Delta P = \eta L 2^{2n+1} R^{-(3n+1)} \left(Q \frac{3n+1}{\pi n}\right)^n \quad \text{(B)}$$

If the circular untapered (cylindrical) die is modified by tapering so that it becomes convergent and takes the shape of a truncated right cone, one has to consider the pressure drop dP for an infinitely small length dl in terms of dr and integrate with respect to r between the limits of R_1 and R_2, remembering that $dl = f(dr)$, namely,

$$dl = -dr \cot\theta$$

$$dP = \frac{\eta dl 2^{2n+1}}{r^{3n+1}} \left(Q \frac{3n+1}{\pi n}\right)^n,$$

$$\Delta P = -2^{2n+1} \eta \cot\theta \left(Q \frac{3n+1}{\pi n}\right)^n \int_{R_1}^{R_2} r^{-(3n+1)} dr$$

$$= -2^{2n+1} \eta \cot\theta \left(Q \frac{3n+1}{\pi n}\right)^n \left(-\frac{1}{3n}\right) r^{-3n} \Big|_{R_1}^{R_2}$$

Taking the limits and simplifying,

$$\Delta P = \frac{2^{2n+1}}{3n} \eta \cot\theta \left(Q \frac{3n+1}{\pi n}\right)^n R_2^{-3n} \left[1 - \left(\frac{R_1}{R_2}\right)^{-3n}\right] \quad (4)$$

At this stage we wish to draw attention to the symbol η in the various equations. The viscosities in wide-slit dies and in dies with cylindrical and conical geometries are not, strictly speaking, identical. Rewriting them η'' and η', respectively, one may take advantage of the empirical relationship between them which was used by Carley,[3] namely,

$$\eta'' = 0.91\eta'$$

Having thus identified η in eqs. (B) and (4) with η', we can therefore replace the η of eqs. (A) and (1) through (3) with 0.91 η'. This makes it possible to apply viscosity data obtained from rheometers with circular channels to the calculation of pressure drops in wide-slit dies.

DISCUSSION

It is clear that the method of calculating pressure drops may also be applied to dies in which the taper is curvilinear rather than rectilinear. Such dies are probably more expensive to produce, but they would have the considerable advantage of giving rise to a minimum of flow disturbance. The kind of die that would probably be near ideal would have a conicocylindrical funnel shape such as may be generated by rotating an odd-power parabola around an axis which is practically parallel with the positive and negative branches of the parabola some sufficient distance r away from its point of inflexion; it is clear that r here defines the radius of the circular cross section of the resulting channel at any point between the entrance (R_1) and the exit (R_2). Such a die would minimize the flow discontinuity at its entrance where the flow transition from a broad and sluggish stream to a narrow and fast stream occurs. The optimum gradualization of this transition will have obvious benefits by enabling the extruder increase the pressure, and hence the output without exceeding the limit at which serious extrudate defects are encountered when ordinary dies are used.

In order that the pressure drop in such an advanced die may be calculated, it would merely be necessary to determine the function of dl in terms of dr, followed by substitution in an equation for dP based upon eq. (B) and integration between the limits of R_1 and R_2 in the same manner as that shown above.

CONCLUSIONS

A simple method for deriving flow equations for tapered dies is given. This requires the identification of the flow geometry, specifically whether the cross section is wide-slit or circular, and a knowledge of the function which defines the cross-sectional parameter(s) in terms of the channel length at any point along the principal flow axis between the die entrance and the die exit. Given such a flow equation, it is possible to optimize die design for maximum output at acceptable extrudate quality.

References

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2. O. Plajer, *Plastverarbeiter*, **23**, 407 (1972).
3. J. F. Carley, *SPE J.*, **19**, 977 (1963).

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